NAG Fortran Library Routine Document

S09ABF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S09ABF returns the value of the inverse circular cosine, $\arccos x$, via the routine name; the result is in the principal range $(0, \pi)$.

2 Specification

```
real FUNCTION S09ABF(X, IFAIL)
INTEGER              IFAIL
real                 X
```

3 Description

The routine calculates an approximate value for the inverse circular cosine, $\arccos x$. It is based on the Chebyshev expansion

$$\arcsin x = x \times y(t) = x {\sum_{r=0}}' a_r T_r(t)$$

where $\dfrac{-1}{\sqrt{2}} \leq x \leq \dfrac{1}{\sqrt{2}}$, and $t = 4x^2 - 1$.

For $x^2 \leq \dfrac{1}{2}$, $\arccos x = \dfrac{\pi}{2} - \arcsin x$.

For $-1 \leq x < \dfrac{-1}{\sqrt{2}}$, $\arccos x = \pi - \arcsin \sqrt{1 - x^2}$.

For $\dfrac{1}{\sqrt{2}} < x \leq 1$, $\arccos x = \arcsin \sqrt{1 - x^2}$.

For $|x| > 1$, $\arccos x$ is undefined and the routine fails.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

5 Parameters

1: X – ***real*** *Input*

On entry: the argument x of the function.

Constraint: $|\mathrm{X}| \leq 1.0$.

2: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

S09ABF has been called with |X| > 1.0, for which arccos is undefined. A zero result is returned.

7 Accuracy

If δ and ϵ are the relative errors in the argument and the result, respectively, then in principle

$$|\epsilon| \simeq \left| \frac{x}{\arccos x \sqrt{1-x^2}} \times \delta \right|.$$

The equality should hold if δ is greater than the ***machine precision*** (δ is due to data errors etc.), but if δ is due simply to round-off in the machine it is possible that rounding etc. in internal calculations may lose one extra figure.

The behaviour of the amplification factor $\dfrac{x}{\arccos x \sqrt{1-x^2}}$ is shown in the graph below.

In the region of $x = 0$ this factor tends to zero and the accuracy will be limited by the ***machine precision***. For $|x|$ close to one, $1 - |x| \sim \delta$, the above analysis is not applicable owing to the fact that both the argument and the result are bounded $|x| \leq 1$, $0 \leq \arccos x \leq \pi$.

In the region of $x \sim -1$ we have $\epsilon \sim \sqrt{\delta}$, that is the result will have approximately half as many correct significant figures as the argument.

In the region $x \sim +1$, we have that the absolute error in the result, E, is given by $E \sim \sqrt{\delta}$, that is the result will have approximately half as many decimal places correct as there are correct figures in the argument.

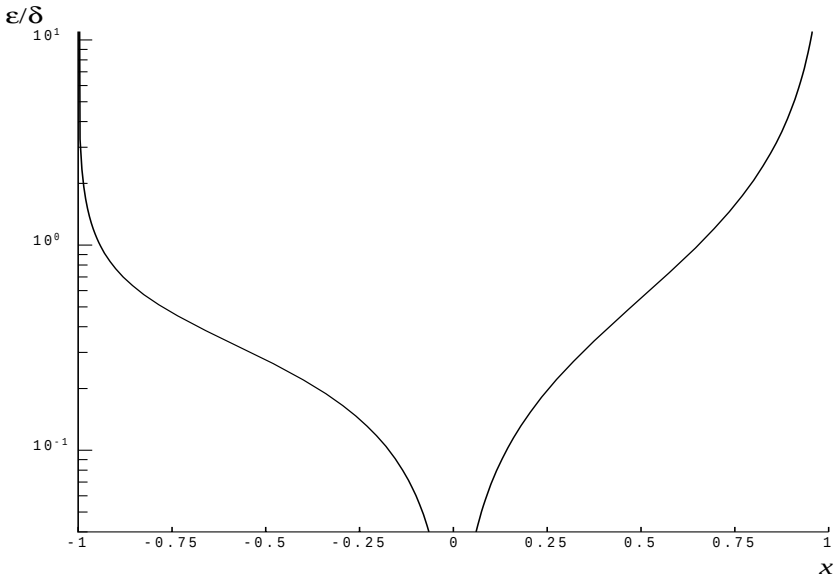

Figure 1

8 Further Comments

None.

9 Example

The example program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S09ABF Example Program Text
*     Mark 14 Revised.  NAG Copyright 1989.
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             X, Y
      INTEGER          IFAIL
*     .. External Functions ..
      real             S09ABF
      EXTERNAL         S09ABF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S09ABF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*) '     X           Y        IFAIL'
      WRITE (NOUT,*)
   20 READ (NIN,*,END=40) X
      IFAIL = 1
*
      Y = S09ABF(X,IFAIL)
*
      WRITE (NOUT,99999) X, Y, IFAIL
      GO TO 20
   40 STOP
*
99999 FORMAT (1X,1P,2e12.3,I7)
      END
```

9.2 Program Data

```
S09ABF Example Program Data
            -0.5
             0.1
             0.9
             2.0
            -1.5
```

9.3 Program Results

```
 S09ABF Example Program Results

      X           Y        IFAIL

   -5.000E-01   2.094E+00      0
    1.000E-01   1.471E+00      0
    9.000E-01   4.510E-01      0
    2.000E+00   0.000E+00      1
```

```
  -1.500E+00   0.000E+00      1
```
